

FIVE RISK FACTORS MODEL: PRICING SECTORAL PORTFOLIOS IN THE BRAZILIAN STOCK MARKET

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ABSTRACT

The assets risk premium is the central variable of the finance models that seek to estimate the cost of capital of the companies, the cost of this employee, for example, in the evaluation of the stock price. There are several models used to calculate the risk premium, with Fama and French models being widely known and widely disseminated. In 2015, Fama and French introduced a new model with the introduction of two new risk premiums. Due to the relevance of the theme and the possibility of obtaining new information from this new model, the objective of this paper is to conduct a study in the Brazilian stock market from a sample composed of companies listed on the São Paulo Stock Exchange (BMF&Bovespa), testing the ability of sectoral pricing in the risk factors present in the recent 5-factor model, proposed by Fama and French (2015a). In order to carry out the research, the companies listed on the Bovespa were used between January 2008 and December 2015. The results point to a greater importance of the investment risk premium, being statistically significant in three of the five sectors of the economy studied.

Keywords: Pricing Model. 5-Risk Factors. Brazilian Stock Market. Sector Portfolios. SUR Regression.

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1 INTRODUCTION

"Wall Street: money never sleeps". This jargon is not just a movie name, but a reflection of the constant fluctuations in which stock market prices daily pass. In order to understand the factors that give rise to returns, investors and academics seek to achieve for decades the key elements that drive the stock market.

The technical and fundamentalist analyzes are the tools used daily by traders, assets management, institutional funds, and agents who are at the forefront of the negotiations, in an attempt to interpret the oscillations and to beat the market.

On the other hand, academics strive to understand the factors that explain the return of shares. In this way, we seek to formulate a pricing model, capable of reflecting the key factors that explain the market return with the highest possible precision.

In the literature, after CAPM, the most widespread model was the multifactorial model of Fama and French (1993) that selects the risk factors as elements of the pricing model. Risk factors can be defined as risk exposures reflected in the returns of a particular asset class.

In this context, risk factor models have been used not only in the academic field, but also by quantitative funds that try to capture excess returns from risk factors present in the business and macroeconomic environment. The risk-based investment strategy seeks to identify drivers of returns. In the macroeconomic field one, can cite as relevant factors: economic growth, real interest rate, inflation, default rate, country risk. Already in the business environment, liquidity, market value, leverage, in addition to several indices obtained from accounting data.

The four-factor model developed by Carhart (1997) succeeded the Fama and French 3-factor model and originates from the momentum strategy of Jegadeesh and Titman (1993). Such strategy arises from the finding of the existence of abnormal returns positive for a strategy based on the selection of roles, having as parameter the past performance. The strategy establishes a sold position for papers with recent low performance and purchased from those coming from a recent high rally or accumulation of earnings in the last 12 months.

Following this line of studies, Fama and French (2006) derive from the dividend model, discounting the influence of book to market (B/M - relation between the book value and the market value) on the capturing of returns. The main premise of the 5-factor model proposed by Fama and French (2015a) is that the present value of a given stock is calculated from the dividends expected in future financial years, testing the robustness of this model in later works (Fama & French, 2014, 2015b).

The objective of the study is to conduct a study in the Brazilian stock market based on a sample composed of companies listed on the São Paulo Stock Exchange (BMF&Bovespa), testing the ability of sectoral pricing of risk factors present in the recent 5-factor model, proposed by Fama and French (2015a).

It is important to emphasize that the idea of the present study is not to replicate the original 5-factor model of Fama and French (2015a), but to use the five risk factors proposed by the authors in the composition of a pricing model, through a methodology adapted to the conditions of the Brazilian capital market.

From the selection of the sample and calculation of the risk factors, we obtained the pricing models to be applied to sectoral portfolios assembled year by year. The idea is to show the model's capacity to price the average weekly returns of the main sectors of activity of the companies listed on Bovespa (basic materials, cyclical consumption, non-cyclical consumption, industrial and public utility).

2 LITERATURE REVIEW

Markowitz's portfolio selection theory (1952) was the initial step in the development of various asset pricing models that seek to determine the expected return. The central hypothesis is that investors make their decisions backed by two parameters present in the probability distributions of assets: the mean and the variance. These metrics refer to the classic risk-return relationship, always inherent to the process of choosing an investment. For this reason, Markowitz's framework is also known in the literature as a mean-variance model.

Considering the assumptions of this theory, to compose an optimal investment portfolio, the economic agent should allocate its resources to the portfolio that presents the smallest variance among an infinite set of portfolios that bring a certain expected return (Caldeira, Moura, & Santos, 2013).

In this context, the return on an investment portfolio can be measured by means of the individual expected returns of the component assets, weighted by their respective portfolio weights (Investment Science p.166). The variance of the portfolio can be obtained from the individual variances of the assets.

Diversification is a basic principle of this theory. It is considered capable of mitigating non-systematic risk (of the companies themselves), whenever the assets do not have a perfect correlation with each other (other than $\rho = 1$).

Systematic (market) risk, in turn, cannot be eliminated, but can be optimized through the portfolio of minimum variance, which generates the lowest risk level for the investor.

In the stock valuation models, the asset risk premium is the central variable in explaining the return of these assets. In this context, two lines of research stand out in the improvement of the study of the CAPM model. One of these lines of research focused on the dynamic treatment of the model, in which risk factors vary over time. These models are called conditional CAPM. In these models, it is not assumed that the relationship between the return of the asset and the risk factor is static. It seeks to reflect the changes that occur in the market over time.

The other line of research sought to study multiple risk factors capable of explaining the return of the target asset. The focus was on what organizational factors would be able to explain the company's performance. It is in this line that the present research fits and seeks to apply the most recent model of Fama and French, the 5-factor model. In sections 2.1 to 2.3, the evolution of the uni factor model (CAPM model) to the current multi-factor model (5-factors) will be explored.

2.1 Capital Asset Pricing Model

The portfolio theory was the basis for the works of Sharpe (1964), Lintner (1965) and Mossin (1966), who formulated the CAPM (Capital Asset Pricing Model). The Sharpe-Lintner version is the most widespread in the literature, establishing that the expected return of the asset $E(R_i)$ is equal to the sum of the risk-free rate, R_f , with the risk premium of the asset, $\beta_{im}[E(R_m) - R_f]$. $E(R_m)$ is the expected return on the market portfolio (systematic risk).

The CAPM parameter β_{im} is the core of the model and seeks to capture the sensitivity of the asset to market portfolio oscillations. It is calculated by the quotient between the active-market covariance and the variance of the market return.

Therefore, insofar as the systematic risk cannot be mitigated, investors would theoretically be compensated for by higher returns by carrying higher risk embedded portfolios, in turn captured by the beta sensitivity of the market.

Another well-known version is Black, known as Zero Beta CAPM. The difference in relation to the previous strand is the substitution of the risk-free rate for the return of a portfolio Z, with no correlation with the market portfolio.

The result of the Zero Beta model is interesting. It strengthens the CAPM insofar as it shows that the efficient market portfolio, in terms of mean-variance, can be achieved not only from risk-free loans (CAPM Sharpe-Lintner's premise), but also from the sale to Risk-bearing assets (Fama & French, 2004).

CAPM, in its various versions, is built on the so-called efficient market hypothesis, encompassing, as premises, the elimination of diversifiable risk (through the construction of a portfolio with minimal variance), inexistence of transaction costs, symmetric information, rational investors averse to risk, and market equilibrium, among others.

Despite the inability of the premises to reflect the natural conditions of the market, CAPM served (and still serves) as a theoretical analytical-base model for the construction of more complex models.

The CAPM postulates that the return of an action can only be explained by the beta parameter. However, since the 1980s, empirical evidences contrary to the explanatory capacity

of the market beta appear in the literature, such as the work of Stambaugh (1982) and Fama and French (1992). Such studies show that the beta-return relationship is less pronounced (or more horizontal) than the predicted by the Sharpe-Lintner model.

It is important to highlight Roll's critique (1977) of the impossibility of testing the CAPM model, insofar as the true market portfolio, with all the marketable assets present in the economy, cannot be replicated.

2.2 The 3-factor model of Fama and French

In the search for a pricing model with greater scope and ability to explain the return of papers listed on NYSE, AMEX and NASDAQ, Fama and French (1992) elaborated a 3-factor model: market risk, size and B/M ratio.

The sample consists of shares of the three Stock Exchanges between 1963 and 1991. To calculate the model factors, Fama and French (1992) constructed portfolios to replicate the Small Minus Big (SMB) factor, defined by the market capitalization of the companies, as well as the book market (High minus Low - HML) factor. The excess market return, $R_{m,t} - R_{lr,t}$, present in the single factor model, will also compose the model:

$$R_{i,t} - R_{lr,t} = a_i + b_i(R_{m,t} - R_{lr,t}) + s_i(SMB_t) + h_i(HML_t) + e_{it} \quad (1)$$

Wherein $R_{i,t}$ is the return of portfolio i in month t ; $R_{lr,t}$ is the return of the asset with risk free rate in month t ; $R_{m,t}$ is the return of the market portfolio in month t ; SMB_t is the premium for the size factor in month t ; HML is the premium for the B/M factor in month t ; e_{it} is the error term of the model.

The authors find evidence of positive premiums for the three risk factors; the regression of the model portfolios presents an intercept statistically equal to zero, which validates the risk factors such as proxies for the pricing model. The model was able to neutralize problems of multicollinearity between the factors, besides showing superior explanatory capacity to the CAPM for the returns of shares in the American financial market.

Another interesting result, B/M has explanatory power for the average returns greater than the size effect. Dividing the sample into 12 portfolios, based on B/M, companies with lower B/M obtained an average return of 0.3%, while those with the highest index reached 1.83% of return in the period analyzed. The explanation for this phenomenon is based on the fact that shares with high B/M are considered riskier than those with reduced B/M. According to Fama and French (1992), the highest return would be a way to compensate for the greater risk borne by high B/M papers.

2.3 The 5-factor model of Fama and French

Fama and French (2006) derive from the dividend model, discounting the influence of B/M on the capture of returns. The main premise of this model is that the present value of a certain share is calculated from the dividends expected in future financial years.

M_t is the sum of the dividends discounted by i , the internal rate of return (IRR) of the expected dividends, supplying the share price in period t . By means of an accounting derivative, they arrive at the value of the share as: the return per share, RPA , less the change in the net book value per share, ΔB_t .

$$M_t = \sum \frac{E(RPA_{t+1} - \Delta B_{t+1})}{(1+i)^t} \quad (2)$$

$$\Delta B_t = B_t - B_{t-1} \quad (3)$$

Dividing the present value of the dividends (which in theory must correspond to the market price of the share, M_t) by the value of the net worth account is the inverse of the B/M metric:

$$\frac{M_t}{B_t} = \frac{\sum \frac{E(RPA_{t+1} - \Delta B_{t+1})}{(1+i)^t}}{B_t} \quad (4)$$

The derivation from this cash flow allows the following inferences: *ceteris paribus*, a lower value of M_t (higher B/M) implies higher expected return; maintaining fixed B_t , M_t and the expected profit, the increase in the balance sheet growth (investment, asset expansion) generates lower expected returns.

The importance of this valuation model is to provide factors that, included in the pricing model, can efficiently capture the expected returns. In this context, motivated by the works of Novy-Marx (2012) and Aharoni, Grudy and Zeng (2013), who find favorable evidence for the average returns and profitability ratio, and for the average returns and investment ratio, respectively, Fama and French (2015a) added two new parameters to the original three-factor model:

$$R_{i,t} - R_{fr,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (5)$$

The RMW factor is calculated from the operational profitability measured by the company. The financial index that makes this calculation possible is the operating profit/PL ratio. Therefore, the factor measures the return difference obtained by stocks with robust and weak operating performance.

The CMA risk factor is calculated using the total assets variation from year to year as a parameter. The factor idea is to measure the difference in return between companies that expanded their total assets with more intensity (more aggressively) and companies that had more moderate asset expansion, or even had a contraction in their (conservative) asset position.

In a later study, Fama and French (2015b) tested the robustness of this 5-factor model based on anomalies, suggested by the market beta literature, stock repurchases, volatility, accruals and timing. The results show that returns from profitable firms and conservative investment (positive RMW and CMA) capture high average returns associated with low beta market, share repurchases and low volatility.

Testing the 5-factor model in international capital markets, Fama and French (2015c) confirm the existence of a positive relationship of B/M and profitability, and a negative relationship of the investment, with the average returns of the stock markets of North America, Europe and Pacific Asia. The model is also tested for the Japanese market by providing favorable evidence for the explanatory power of the B/M, which does not occur with profitability and investment factors.

3 METHODOLOGY

The present research seeks to investigate if the adapted 5-factor model is able to accurately price the variations of the returns of sectoral portfolios.

The method used to construct the portfolios (used to calculate the risk factors) is the 2x2x2x2 model. This method produces 8 portfolios, segmented by the median of the metrics of the four risk factors (SMB, HLM, RMW, CMA).

Some differences from the Fama and French (2015a) method should be highlighted. Firstly, the authors build their portfolios from monthly value weighted returns. In this work, equally weighted weekly returns are used. The metrics of the risk factors undergo an adaptation, related to the RMW factor, as explained in section 3.1.

The criteria for inclusion in the sample were those adopted by Leite, Pinto and Klotzle (2016), who seek to adapt the sample to the conditions of the Brazilian stock market. The theoretical justifications of the adopted methodology are detailed in section 3.2.

3.1 Sample and Data Collection

The sample was assembled from stocks listed on BMF&Bovespa from January 2008 to December 2015. The choice of period is justified by the large number of initial public offerings (IPOs) that occur between 2006 and 2008. In this sense, the collection of data from 2008 provides the possibility of working with a wider and more robust sample for the pricing model. Between February 2006 and June 2008, the domestic stock market experienced a boom in terms of public listing. During this period, 94 IPOs were authorized by the Securities and Exchange Commission, reflecting the favorable macroeconomic scenario that the Brazilian economy was undergoing.

In the study, weekly return series obtained from the closing prices of the companies included in the sample were used. All data was collected from the Bloomberg terminal, except the 30-day SWAP-DI rates, obtained from the BMF&Bovespa information retrieval system.

The initial selection of the assets to compose the portfolios is carried out year by year, prospectively, at the end of December of each year. In other words, the position of listed companies with active trading at the end of December of each year is used in the initial gross sampling, which will go through the subtracted exclusion criteria. The net sample, obtained after applying the exclusionary criteria, will provide the papers that will compose the portfolios of the subsequent year.

In this context, the sample is redefined from an annual periodicity, based on the following criteria:

- The share must be traded in at least 50% of the trading sessions of the year, liquidity criterion;
- Excluding shares with negative Shareholders' Equity (PL), companies with $PL < 0$ characterize insolvency, being able to bias the HML and RMW factors, which directly use the book value of the PL in its calculation;
- Excluding banks and insurance companies, the composition of the balance sheet and the income statement of the companies of these segments have particular characteristics, requiring a methodology of own evaluation for the sectors. Its inclusion may bias the model.

By applying the aforementioned criteria, the following amounts of assets were obtained year by year: 88 in 2008, 111 in 2009, 197 in 2010, 194 in 2011, 193 in 2012, 175 in 2013, 188 in 2014, and 178 in 2015. By obtaining the filtered assets, these were segmented into portfolios, whose selection criterion was sectorial. The calculation of the risk factors was carried out from the portfolios assembled.

The series of weekly returns from 2008 to 2015 provided the total of 417 observations (weeks) that, through calculations of weekly risk factors, produced 417 data for the estimation of the pricing model.

3.2 Base variables

The adjusted weekly return on shares is used to include dividends. The Bloomberg terminal has a configuration feature that allows embedding mandatory dividends, additional dividends and interest on equity in the return of the shares, as well as to adjust historical stock prices for splits, inplits, increases and capital reductions. Adjustments to distribute dividends and corporate events carried out by the terminal avoid distortions in weekly returns, in addition to embodying the global return measured by the shareholder.

Applied to the nominal return with dividends is the Neperian logarithm for transforming the discrete returns into continuous ones.

$$R_{i,t} = \ln \left(\frac{Div_{i,t} + P_{i,t}}{P_{i,t-7}} - 1 \right) \quad (6)$$

Wherein $P_{i,t}$ is the closing price of the trading day of t and $P_{i,t-7}$ the closing price of the day of the previous week, $t-7$. The quotient of the two variables provides the weekly variation of the stock price.

We adopt the hypothesis of equally weighted weights to calculate the average weekly return of the market portfolio, measured from the shares that met the eligibility criteria.

$$R_{c,t} = \frac{1}{N} \left(\sum_{i=1}^n R_{i,t} \right) \quad (7)$$

Wherein $R_{c,t}$ is the weekly return of the portfolio is at week t , $R_{i,t}$ is the stock return i in week t and N is the number of shares in the portfolio. The choice of adopting equally weighted returns avoids an inherent problem in the Brazilian stock market: the high concentration of trading in a small number of stocks. The low dispersion among stocks in relation to the volume traded is a common feature of capital markets in emerging markets.

The B/M index, calculated on the basis of shareholders' equity, is taken from the balance sheet as of December 31 from $t-1$, and the market value at the same date, December 31 from $t-1$.

$$\frac{B}{M_{i,t}} = \frac{VC_{PL,dezembro(t-1)}}{VM_{dezembro(t-1)}} \quad (8)$$

The market value of a share (VM) is a reflection of the market's expectation for the ability to generate cash flow and to make profits of a company. The net equity (VC_{PL}) is the difference between the book value of the assets and liabilities, it is the residual value between the debt (assets) and the credit (liabilities) of the balance sheet of a company.

Unlike the current strategy of Jegadeesh and Titman (1993), which uses stock performance in the capital market to calculate the WML (Winning Minus Losers) risk factor, the RMW (Robust Minus Weakens) factor seeks to reflect the operational performance of a company, that is, its ability to generate cash flow to the shareholder.

In the work of Fama and French (2015a), the metric used is the operating profit divided by the net equity. Operating income is obtained from net revenue less general and administrative sale expenses. Due to the difficulty in accessing the Brazilian data of these specific accounts of the income statement, an adjustment was made. Operating profit has been replaced by EBIT. The factor is calculated from the financial index EBIT/PL.

$$RMW = \frac{EBIT_{dez(t-1)}}{VC_{PL,dez(t-1)}} \quad (9)$$

This index is similar to ROIC (return on invested capital), a measure that measures the operational capacity of a company to remunerate the capital invested by the shareholder. The ROIC is calculated by the EBIAT/Invested Capital Index. The measure of the denominator is Earnings Before Interest After Taxes (EBIAT) and the divisor is the invested capital (or the paid-up capital stock), an integral part of Net Equity.

This measure reflects the basic productivity of the capital invested in the business (it is the return obtained by each monetary unit applied in the business). The higher the ROIC, the more attractive it is to invest in the company.

At this point, it is interesting to note that two of the risk factors use PL in its composition (HML and RMW). The use of this balance sheet component as an evaluation tool has pros and cons.

Based on the premise that accounting information carries the fundamental qualitative characteristics - relevance and reliable representation - it can be considered that PL is a stable measure, capable of reflecting the generation of value by the organization. On the other hand, it can be influenced by accounting decisions, regarding the measurement of assets and liabilities. As examples, we can cite the procedures adopted to reduce the recoverable value of fixed assets (impairment tests), and the method adopted in the control and evaluation of inventories, among other decisions capable of affecting the accounting result and, consequently, the position of the PL (Damodaran, 2012).

Based on a measure of profitability, the RMW factor is determined by the same fundamentals of the discounted cash flow model: expected growth, business risk (determinant of discount rate) and cash flow generated (measured result). In this sense, firms with higher expected growth, reduced specific risk and greater dividend distribution, *ceteris paribus*, have higher EBIT/PL.

The fourth factor - CMA (Conservative minus Aggressive), also called INV (investment rate) – in turn is calculated based on the change in corporate assets. The calculation of this risk factor is done by the difference in the position of the total assets between the end of year t-1 and the end of year t-2. The idea of the factor is to measure the rate or variation of investment of the companies, reflected in the expansion of assets in the balance sheet.

$$CMA = \frac{A_{dez(t-1)} - A_{dez(t-2)}}{A_{dez(t-2)}} \quad (10)$$

There are two ways for a company to carry out the aforementioned expansion: CAPEX (capital expenditure) – amount of capital expenditure that exceeds depreciation – and the need for capital turnover.

The risk factors mentioned in this section can be interpreted as diversified portfolios, capable of providing different combinations of exposure to the metrics: market capitalization (SMB), B/M index (HML), EBIT/PL index (RMW) and changes in assets (CMA).

3.3 Sectoral portfolios

The sectorial portfolios used in this study are formed from the companies that met the inclusion criteria to calculate the risk factors. The sectors listed were: basic materials, cyclical consumption, non-cyclical consumption, industrial and public utility, defined from the Bloomberg terminal sector filter, following a similar structure to the sector indexes prepared by BM&FBovespa:

- Basic Materials Portfolio - covers segments such as the chemical, pulp and paper, metallurgy, mining and steel industries. It can be considered the pillar of the productive chain, insofar as it supplies the raw materials and inputs for the various fields of productive activity;

- Cyclical Consumption Portfolio - comprised of wholesale, retail, clothing, textile, auto parts and equipment, travel and leisure, hotels and restaurants, real estate developers. This is a sector closely followed by analysts and economists who work with economic conjuncture, considered one of the main thermometers of the level of economic activity;

- Non-Cyclical Consumption Portfolio - includes the food and beverage industry, farming, commerce and distribution, pharmaceutical, tobacco, various services (educational, laboratorial, highways, operational leasing). In contrast to the volatility and seasonality characteristics of the cyclical consumption sector, non-cyclical consumption is characterized by greater homogeneity in terms of revenues;

- Industrial Portfolio - composed of companies that operate in the capital goods industry (machinery and equipment), logistics (services and transportation material) and electrical equipment;

- Public Utility Portfolio - brings together the subsectors for water supply, electricity, gas and sanitation. They are services rendered by means of delegation of the public power that seek to attend to the satisfaction of collective well-being.

Table 1
Spearman correlation between Sectorial Portfolios

	Basic Material	Cyclical consumption	Non-cyclical consumption	Industrial	Public Utility
Basic Material	1				
Cyclical consumption	0,6084	1			
Non-cyclical consumption	0,4255	0,4468	1		
Industrial	0,7836	0,7260	0,7260	1	
Public Utility	0,4252	0,4464	0,9999	0,6633	1

Note. Source: Prepared by the authors.

Table 1 shows the correlation between the weekly returns of the sector portfolios. The highest correlations occur between the industrial and basic materials sectors ($\rho=0.7836$) and between the public utility and non-cyclical consumption sectors ($\rho = 0.9999$). All sectors present medium or high correlation.

The presence of lower correlations by the public utility sector with the returns of the other segments (last line of the table) is justified because it belongs to a particular market structure, as will be explained in section 4.2. Its higher correlation with the non-cyclical consumption sector ($\rho=0.9999$) shows a common characteristic among the sectors: the lower dependence of the level of economic activity insofar as both provide essential services to society, characterized by low elasticity of demand.

The sector premiums are calculated from industry portfolios assembled on the basis of equally weighted returns. The junction of these parameters gives rise to the following regression model:

$$R_{\text{CarteiraSetorial},t} - R_{f,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (11)$$

Section 4 details the results obtained from these regression, estimated by the OLS (Ordinary Least Squares) and SUR (Seemingly Unrelated Regression) methods.

3.4 Construction of portfolios for calculation of risk factors

The base order month is the end of December of each year t , coinciding with the end of the Brazilian fiscal year. In the first stage, the shares are ordered according to their market value at the end of December of year t . This ordering, done in a decreasing sequence, allows the segmentation of the companies listed in two portfolios: B (Big) and S (Small).

The second order uses the B/M index as a parameter, allowing the formation of two new H (High) and L (Low) portfolios. In the third order, the shares are classified according to the EBIT/PL index, segmenting the portfolios in R (Robust) and W (Weak). Finally, in the fourth order, the metric becomes the total change in assets, or the investment rate, leading to the unfolding of the portfolios in C (Conservative) and A (Aggressive). This procedure is repeated eight times at the end of each year of the study period (2008 to 2015).

In all four orders, the parameter used to separate portfolios is the median of the base variables (market value, B/M, EBIT/PL, change in total assets). The median therefore serves as a dividing line for portfolio segmentation. This way of building the portfolios produced eight sets of diversified assets (portfolios) that allowed, each year, the calculation of risk factors, described in the following section.

3.5 Calculation of Risk Factors

The main pricing studies with risk factors, such as Fama and French (1993), use monthly returns to calculate them. On the other hand, they analyze an extended period of time. In the case of this study from 1963 to 1991.

In the present study, given the shorter extension of the investigated period, we opted to work with weekly returns, in order to increase the number of observations and, thus, to measure more accurately the calculation of the risk factors.

The market factor is calculated from the difference between the weekly evenly weighted returns and the weekly risk free rate, R_{lr} , computed from linear interpolation performed from the 30-day SWAP-DI rate. The difference between the two metrics provides the excess return $R_{c,t}$ of the portfolio c at day t::

$$R_{c,t} = \frac{1}{N} \left(\sum_{i=1}^n R_{i,t} \right) - R_{lr} \quad (12)$$

Calculated weekly, from a long position in the portfolio with shares of companies with low market capitalization (Small) and sold in shares of companies of great market value (Big). The difference in the weekly return between the portfolios provides the SMB risk factor.

Calculated weekly, from a long position in the portfolio with shares of companies with a high B/M ratio and sold in shares of companies with a low ratio. The difference in return between the two positions provides the HML risk factor.

Calculated weekly, from a position bought in the portfolio with stocks of companies that obtained robust operating performance (Robust) and sold in shares of companies with weak operating performance (Weak). These performance measures were computed based on the operating profit/PL obtained at the end of each year t-1. The difference in return between the two positions provides the RMW risk factor.

Calculated weekly, from a long position in the portfolio with shares of companies that had reduced investment rates between the financial years (Conservative) and sold in shares of companies that expanded their assets with greater intensity (Aggressive). The difference in return between the two positions provides the CMA risk factor.

Table 2 shows the presence of a low correlation between the risk factors, a positive attribute for the pricing model, insofar as it reduces possible multicollinearity problems among the model variables.

Table 2
Spearman correlation between Sectorial Portfolios

	Rm-Rf	SMB	HML	RMW	CMA
Rm-Rf	1				
SMB	-0,2449	1			
HML	0,2097	-0,0426	1		
RMW	-0,2423	-0,0402	-0,0303	1	
CMA	-0,0884	0,0447	0,0029	0,0473	1

Note. Source: Prepared by the authors.

We highlight the inverse correlation between the market premium and the SMB factor ($\rho=-0.2449$) and the RMW factor ($\rho=-0.22423$), the most intense of the correlation matrix.

4 RESULTS

This section presents the search results. The methods are applied to the returns of the five proposed sectoral portfolios, each one based on the five calculated risk factors. From the sector portfolio premiums used as explained variable and risk factors as explanatory variables, five linear regression are formed, which seek to explain the relationship between sector portfolio returns and risk factors.

The SUR methodology is processed in two stages. In the first step, OLS regression residuals are used to estimate the covariance matrix of the errors of the equations. In a second moment, the coefficients of the regression were estimated via GLS, when the previously estimated covariances were applied (Duarte, Lamounier, & Takamatsu, 2007).

If the error covariance matrix obtained in the first step is zero, the OLS and SUR methods are equivalent. As a non-zero covariance matrix, the SUR method performs the correction of residues, generating greater accuracy in the estimation process (Neves, 1996).

The results obtained by the study corroborate the presence of a non-zero covariance matrix for the pricing model, reflected in the superiority of the R^2 adjusted from regression estimated by the SUR method in comparison to those estimated by OLS, as demonstrated in section 4.2.

The option by the SUR method follows the Costa and Neves (2000) approach, which, applied to the 5-factor model, makes it possible to test the statistical significance of the four risk factors that expand the CAPM model at the same time, as it reflects the capacity of the Adjustments in the market premium of sector portfolios.

4.1 Statistical Analysis

Initially, a preliminary analysis of the data was made from the correlation matrix and graphical visualization of the series of risk factors. The first step was to understand in a preliminary way the interrelationship of variables. In a second moment, the OLS and SUR methods were evaluated (section 4.2).

Table 3

Stationary Testing

Returns	Augmented Dickey-Fuller	p-value
Rm-Rf	-5,3744	<0,01
SMB	-4,9524	<0,01
HML	-6,7011	<0,01
RMW	-6,8917	<0,01
CMA	-6,0431	<0,01

Note. Source: Prepared by the authors.

The Augmented Dickey-Fuller (ADF) test is performed for the five risk factors of the model in order to verify whether the returns are stationary or not. The importance of the test is due to the fact that non-stationary series have a temporal tendency; they may present high explanatory power even if the variables are not correlated (Brooks, 2014).

The results of the stationarity tests are shown in Table 3 for the returns of the risk factors and in Table 6 for those of the sector portfolios. No tested return series presented unit root. They are stationary.

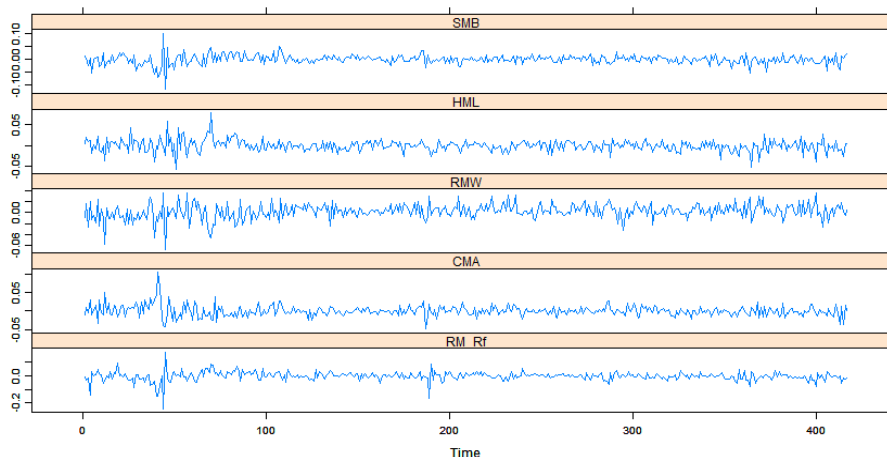


Figure 1. Series of weekly returns of risk factors: Jan-2008 to Dec-2015

Source: Prepared by the authors.

Analyzing the graphs of the behavior of the risk factors, one can observe certain behaviors for the period of analysis. The first of these is the strong acceleration of spreads

volatility caused by the subprime crisis, from the fourth quarter of 2008, remaining until the middle of 2009.

During this period, there is a particular behavior on the part of the CMA factor, when the investors direct their resources to conservative companies, to the detriment of the bold ones. This movement that reflects a process of risk aversion is translated into the positive spread of the CMA factor.

Another interesting scenario, a new acceleration of spreads (less intense than that occurred at the end of 2008) occurs from the end of 2010, notably in the market premium (Rm-Rf) and in the CMA factor, this time drawn by the European debt crisis, promoted by strong budget deficits linked to the rise in public debt in European countries. The factor, however, that effectively triggered the new cycle of volatility was the onset of global market fears with the possibility of default on Greek debt from the second half of 2010. This fear was confirmed years later, more precisely on June 30 2015, with the expiration of the deadline and non-payment of the taxiing of the debt contracted with the IMF (International Monetary Fund).

The third and final widening of the spreads of the series occurs from the last quarter of 2014, starting from the presidential elections in Brazil. The increase in variation is clearly seen in the HML and RMW factors, persisting during the year 2015, as a result of President Dilma's continued presence in power, linked to the macroeconomic scenario of stagflation in the Brazilian economy.

Table 4
Descriptive Statistics of Risk Factors

Statistic	Rm-Rf	SMB	HML	RMW	CMA
Maximum	0,1700	0,1010	0,0763	0,0357	0,1046
Minimum	-0,3422	-0,1206	-0,0565	-0,0665	-0,0499
Average	-0,0086	-0,0025	-0,0009	0,0019	0,00004
Medium	-0,0029	-0,0016	-0,0005	0,0028	-0,0009
Standard deviation	0,0489	0,0183	0,0058	0,0135	0,0145
Asymmetry	-3,72	-0,49	0,29	-0,69	1,15
Kurtosis	24,12	9,78	6,94	5,59	10,40
Jarque-Bera	<0,01	<0,01	<0,01	<0,01	<0,01
	-5,69	-7,32	-6,44	7,10	362,5

Note. Source: Prepared by the authors.

Descriptive statistics of risk factors (Table 4) show the presence of right asymmetry for the Rm-Rf, SMB and RMW factors, and left asymmetry for the CMA. The HML factor, on the other hand, presents more symmetrical returns. Asymmetry to the right of the market premium is expected, given the combination of low performance of the Brazilian stock exchange, tied to high interest rates; for the RMW factor, the opposite was expected. In relation to the CMA, what is expected is that the low profitability of the capital market in a macroeconomic context that is unfavorable to leveraged companies generates a superior performance on the part of the conservative organizations.

The market factor is the most volatile ($\sigma=0.0489$), the other factors have lower volatilities, all below 2%. The coefficient of variation (CV) was also presented, embodied in the ratio between standard deviation and the average, as a way of measuring volatility. The Jarque-Bera test rejects the null hypothesis of normality for the risk factors.

The ADF test is also performed for the variables explained. The sectoral portfolio premiums do not have a unit root, as shown in Table 5. The analysis of unit root existence is important to verify the possibility of modeling through regression. The existence of a unit root would make this methodology impossible.

Table 5
Stationary Testing

Returns	Augmented Dickey-Fuller	p-value
Basic Mat.	-6,1297	<0,01
Cyclic Cons.	-5,2204	<0,01
Non-Cyclic Cons.	-5,7654	<0,01
Industrial	-5,7271	<0,01
Public Utility	-6,3722	<0,01

Note. Source: Prepared by the authors.

Figure 2 shows the return series of sectoral premiums. Interestingly, the behavior of the cyclical consumer sector premiums during the second shock (2011) is more intense than that of the other sectors. This phenomenon can be explained by the withdrawal of the countercyclical fiscal stimulus granted after the 2008 crisis coupled with a contractionary monetary policy in an attempt to contain the acceleration of the inflation rate and appreciation of the real after a year of robust economic growth (in 2010 the Brazilian GDP grew 7.5%).

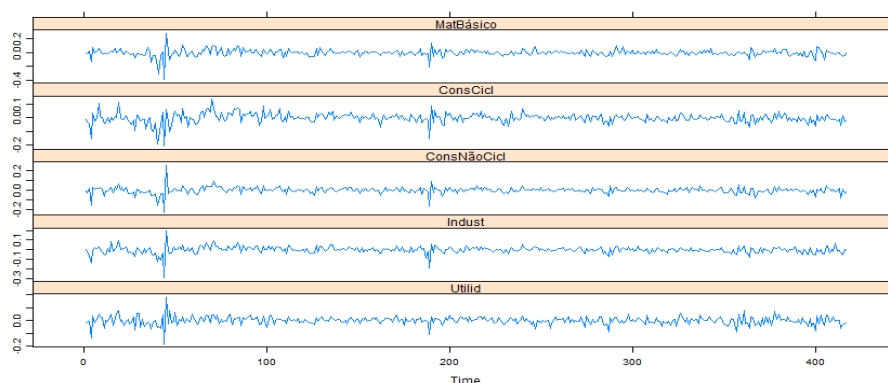


Figure 2. Weekly returns series of sector portfolios: Jan-2008 to Dec-2015

Source: prepared by the authors.

Figure 2 also reflects the volatility of sector premiums brought by the presidential race. China's macroeconomic deceleration scenario, domestic inflationary pressures, rising interest rates, strong depreciation of the real coupled with the beginning of a fiscal adjustment process by the government affected the purchasing power of Brazilians and, consequently, the level of consumption. The sectors of cyclical consumption and basic materials clearly reflected the effects of this adverse scenario.

Table 6
Descriptive Statistics of Sectoral Portfolio Premiums

Statistic	Mb	Cc	Cnc	Ind	UtP
Maximum	0,2816	0,1381	0,2582	0,1927	0,1791
Minimum	-0,3903	-0,2061	-0,2235	-0,2971	-0,1832
Average	-0,0034	-0,0023	-0,0016	-0,0032	-0,0009
Medium	-0,0027	-0,0007	-0,0002	-0,0007	0,0005
Standard deviation	0,0487	0,0358	0,0329	0,0348	0,0304
Asymmetry	-1,48	-1,08	-0,39	-1,61	-0,45
Kurtosis	18,68	9,57	19,60	19,28	9,55
Jarque-Bera	<0,01	<0,01	<0,01	<0,01	<0,01
	-14,32	-15,56	-20,56	-10,87	-33,78

Note. Source: Prepared by the authors.

The descriptive statistics of sector portfolios (Table 6) demonstrates the presence of higher volatility of weekly returns by the portfolio of commercial materials ($\sigma=0.0487$) followed by cyclical ($\sigma=0.0358$) and industrial ($\sigma=0.0348$) consumption, which makes sense insofar as they are more sensitive to fluctuations in the level of economic activity (sectors with higher price

elasticities and demand income). We also highlight the right asymmetry of returns of all sectors (common characteristic in the stock market), more accentuated in non-cyclical consumption and public utility. The Jarque-Bera test rejects the null hypothesis of normality of the weekly premiums for all sector portfolios.

4.2 Regression

In this section, the parameters obtained from regression estimated by the SUR method will be presented. The economic package used for the estimation process was R Studio. For correlation of the regression estimates, the Newey-West covariance matrix (HAC matrix) was applied.

The results obtained by the OLS method were not presented to save space and not to interfere with the reader (and can be sent to interested parties). The comparison between the results of the two methods of estimation demonstrated the reduction of the standard errors obtained by the SUR method, also reflected in the R^2 empirically adjusted superior to that obtained by means of the OLS method.

The return of the first sectoral portfolio, of commercial materials (Table 7), points to the market price and RMW as statistically significant factors at the 1% significance level. It is interesting to note the relevance of profitability (RMW) in the portfolio price of commercial materials, a volatile sector in terms of results (a negative variation of 1% in the RMW causes a positive oscillation of 0.44% in the return of the portfolio of commercial materials). Its activity is basically concentrated in the circulation of commodities, with low value added, dependent on the international prices of these raw materials and inputs, a factor that impacts the profitability of these organizations. Hedge operations in future markets are usually carried out by the treasuries of these companies, an attempt to mitigate this volatility. The significant 5% intercept provides a caveat for the application of the model in this sector.

Table 7
Basic Materials SUR Regression

$$R_{MB,t} - R_{lr,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Variables	Coefficient	Standard-Error	Statistic t	p-value
Intercept	0,00286	0,00123	2,57	0,0104**
Rm-Rf	1,23896	0,04321	14,82	0,0000***
SMB	-0,09243	0,07001	-0,68	0,4938
HML	0,08706	0,09562	0,74	0,4567
RMW	-0,44131	0,09671	-2,87	0,0043***
CMA	0,22540	0,09008	1,27	0,2063

Note: R^2 adjusted = 0.7492. Significance: ***, ** and * correspond to 0.01, 0.05 e 0.1, respectively.

Note. Source: Prepared by the authors.

In the second sectoral portfolio, the cyclical consumption (Table 8), the statistically significant factors were market premium, RMW and CMA. It should be noted that for this portfolio, as well as for non-cyclical consumption, the intercept was statistically significant at 1%, with a caveat for the model applied in these sectors, which is contrasted by the high R^2 adjusted from the respective models, 86.29% and 87.94%, respectively.

Table 8
Cyclic Consumption SUR Regression

$$R_{CC,t} - R_{r,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Variables	Coefficient	Standard-Error	Statistic t	p-value
Intercept	0,00241	0,00066	3,40	0,0007***
Rm-Rf	1,02146	0,02344	31,44	0,0000***
SMB	0,14476	0,03798	1,64	0,1026
HML	0,08921	0,05187	1,15	0,2514
RMW	0,20092	0,05246	2,18	0,0298**
CMA	-0,14537	0,04886	-2,32	0,0207**

Note: R^2 adjusted = 0.8629. Significance: ***, ** and * correspond to 0,01, 0,05 e 0,1, respectively.

Note. Source: Prepared by the authors.

In the third sectoral portfolio, of non-cyclical consumption (Table 9), the significant factors were market premium, SMB and HML. The SMB and HML coefficients show that a small negative variation in the returns of these factors (-0.12% and -0.18%) causes a positive change in the non-cyclical consumption portfolio (+ 1%).

Table 9
Non-Cyclic Consumption SUR Regression

$$R_{CNC,t} - R_{r,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Variables	Coefficient	Standard-Error	Statistic t	p-value
Intercept	0,00236	0,00057	4,17	0,0000***
Rm-Rf	0,93951	0,02020	48,18	0,0000***
SMB	-0,11829	0,03272	-2,00	0,0466**
HML	-0,18154	0,04469	-3,44	0,0006***
RMW	0,08087	0,04520	-1,04	0,2957
CMA	0,07363	0,04210	1,27	0,2036

Note: R^2 adjusted = 0.8794. Significance: ***, ** and * correspond to 0,01, 0,05 e 0,1, respectively.

Note. Source: Prepared by the authors.

In the fourth sectoral portfolio, the industrial one (Table 10), the relevant factors were market premium and CMA. The significant CMA factor points out a positive influence of the investment rate on the returns of the industrial sector (a variation of 0.24% in the CMA causes a 1% oscillation in the return of the industrial portfolio). This relationship makes sense insofar as the sector is formed by companies with a high asset position, capital intensive, whose main activity is the circulation of capital goods, justifying the intense sensitivity of the premiums of the industrial portfolio in relation to the variation of assets (CMA).

Table 10
Industrial SUR Regression

$$R_{IND,t} - R_{r,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Variables	Coefficient	Standard-Error	Statistic t	p-value
Intercept	0,00138	0,00076	2,19	0,0287**
Rm-Rf	1,01778	0,02667	17,80	0,0000***
SMB	0,07131	0,04321	0,90	0,3685
HML	-0,03829	0,05902	-0,59	0,5558
RMW	0,09663	0,05970	1,08	0,2797
CMA	0,23623	0,05560	2,26	0,0242**

Note: R^2 adjusted = 0.8135. Significance: ***, ** and * correspond to 0,01, 0,05 e 0,1, respectively.

Note. Source: Prepared by the authors.

In the fifth sectoral portfolio, of public utility (Table 11), the significant factors were market premium, SMB and CMA. The SMB factor, significant at 1%, has an inverse ratio of 0.42% with the return of the public utility portfolio. This behavior has an intuitive explanation, which stems from the market structure of the public utility sector based on the presence of

barriers to entry due to legal regulation (inherent in the concession process for the exploitation of public utility services) and the production process of economies of scale), making it close to a monopoly structure. This feature reflects the influence of the size factor (SMB) on the utilities sector pricing.

Table 11
Public Utility SUR Regression

$$R_{UP,t} - R_{lr,t} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

Variables	Coefficient	Standard-Error	Statistic t	p-value
Intercept	0,00132	0,0008	1,57	0,0922*
Rm-Rf	0,76510	0,0297	15,63	0,0000***
SMB	-0,42006	0,0482	-7,03	0,0000***
HML	0,09723	0,0658	1,37	0,1720
RMW	0,10862	0,0665	1,43	0,1531
CMA	0,25778	0,0620	2,57	0,0106**

Note: R^2 adjusted = 0.6951. Significance: ***, ** and * correspond to 0,01, 0,05 e 0,1, respectively.

Note. Source: Prepared by the authors.

The market premium is statistically relevant in all estimated regression, which corroborates its contribution to the explanatory capacity of the 5-factor model, strengthening the market beta as the most relevant component of the pricing model.

5 CONCLUSIONS

The present work used the components of the 5-factor risk model proposed by Fama and French (2015a) to verify their respective influences on Brazilian stock market returns. The investigated asset class was the weekly returns of sector portfolios.

In order to obtain the components of the pricing model, the 2x2x2x2 portfolio method was used, which produced eight portfolios, based on the variables inherent to the risk factors (market value, B/M, EBIT/PL and asset variation). The ordering was carried out in four steps, producing at each step two portfolios, obtained by a cutting methodology that uses the median of the four factors mentioned above. Having the weekly returns equally weighted from these portfolios, the risk factors are obtained by means of a simple difference of averages.

In the last step, the model was tested. The chosen estimation process was the multiple linear regression of the SUR type. The results point out the importance of the statistically significant market premium for all five sectors tested. In addition to the expected significance of this premium, the risk premium tied to investments was significant in three of the five sectors of the economy studied. This shows the ability to explain the amount of investment in relation to the return of the companies. This fact in particular shows managers how important it is to reflect on the organization's investment policy due to its impact on results.

It is interesting to note in the results that an increase in investments does not always have a positive impact on the performance of the company at that time, a fact verified in the negative coefficient of the cyclical consumption sector, unlike the Industrial and Public Utility sectors, which had positive coefficients.

The presence of significant intercepts, at a significance level of 1% for the consumption sectors (cyclical and non-cyclical) and 5% for those of basic and industrial materials, and of 10% for public utility shows the existence of influences in the returns of sector portfolio premiums not captured by the 5-factor model.

It contrasts with this result, as favorable points to the pricing model, the low correlation between the risk factors (avoiding possible problems of multicollinearity) and the high explanatory power of the factor model reflected in the R^2 adjusted from the respective regression.

One of the limitations of the study is the size of the Brazilian stock market, which makes it difficult to construct diversified portfolios for periods prior to 2008, before the IPO boom

mentioned in section 3.1. The number of listed companies and the low volume of trading in the stock market are also reflected in the difficulty of implementing other methodologies for portfolio construction. In their study, Fama and French (2015a) set up 2x4x4 (32 portfolios), 5x5 (25 portfolios) portfolios, segmenting portfolios from quartiles and quintiles to control variables, for example, difficult replication techniques for the Brazilian market. Therefore, the relevance of adapting the pricing models to the conditions of the capital market investigated is emphasized.

The estimation process is also a limitation. It is recommended that future research uses data in panels, based on the use of time series conjugated with cross-section. The use of the GRS statistical test, adopted in the study by Fama and French (2015a), may be an econometric technique used to validate pricing models that adopt panel data as the estimation method.

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