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# ECONOMIC ASSESSMENT OF FARE REVENUE OF BRAZILIAN METRO-RAIL COMPANIES

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## ABSTRACT

Metro-rail transport is fundamental for urban mobility, and there is a recognition of the need for a subsidy so it can operate efficiently from a social point of view. Thus, this research aims to analyze, through cost functions, the economic viability of fare revenue of metro-rail transport companies. Methodologically, eight companies representing 90% of the passengers transported in the Brazilian metro-rail system were used as references for the capture of costs per passenger and estimation of the mean cost functions per Ordinary Least Squares. The results indicate the possibility of gains with better occupation of the scale and level of efficiency. However, these do not enable operations with the fares practiced by public companies. These results shed light on the management of public fare policies, justifying this research.

**Keywords:** Economy of scale. Cost function. Metro-rail transport.

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## 1 INTRODUCTION

Public transport is fundamental for urban mobility, which gives it significant relevance in terms of city infrastructure and generates a wide range of positive externalities for the urban environment (Xu et al., 2018). Among the various topics discussed in this area are operating costs and fare prices. These themes become even more accentuated in metro-rail transport since most of them receive subsidies to cover their operational costs, leading to a dispute in public choice as to who should bear the transport costs, adding to this issue the effect of the subsidy on efficiency. For example, the state-owned Beijing subway, which has its operation granted to semi-private companies, receives subsidies based on transport costs, safety level, and quality of operations, a sense that this methodology, according to Bai et al. (2012), was able to encourage operators to reduce their costs while maintaining quality and safety levels. However, Obeng's (2019) study found that only 50% of public transport system companies in the United States are cost-efficient and that incentive regulation increases technical inefficiency and diverts companies' costs.

In Brazil, municipal bus systems generally have an economic balance. In contrast, metro-rails depend on subsidies (Vasconcellos et al., 2011). Operator financing is heavily based on fare revenues, significantly different from the European model, where subsidies cover most costs.

Concerning the Brazilian metro-rail transport, it is composed of 15 operators in four types of systems: Subway, Urban Trains, Monorails, and LRTs (Associação Nacional de Transportes Públicos [ANTP], 2020). The metro-rail transport system is classified as small, medium, and large based on the number of passengers transported/year (ANTP, 2020). Medium and large systems are responsible for 98% of passengers transported on the Brazilian metro-rail system, with five systems operated by private companies and six by public companies (ANTP, 2020).

Metro-rail transport is crucial for low-income people because they need to live away from urban centers due to the value of real estate (Rosa, 2006). Moreover, by allowing transport over long urban and often interurban distances, with a relatively low fare, with little travel time and based on electricity, it generates positive externalities such as reduction of pollution, traffic congestion, and all the elements associated with the decrease of this congestion, such as slowness of other means of transport, reduced traffic accidents, reduced noise levels and stress, among others. Thus, its relevance appears far beyond the number of people transported (Bittencourt & Brizon, 2011; Tischer, 2018).

However, this mode of transport has been characterized by being deficient, requiring subsidies to balance the accounts, which compromises public budgets and affects the population as a whole. In this context, the costs for maintaining the operational activities of public transport gain relevance, as they significantly impact transport financing models (Kiggundu, 2009). It is also necessary to highlight that the definition of the level of subsidy to the financing of public transport and, consequently, the definition of the fare price to be charged to the user is a public choice that strongly impacts society. Inadequate subsidy levels can render transport systems inefficient (Arcier, 2014), while charging high fares can limit access to transport, especially for the low-income population (Ševrović, Brčić & Kos, 2015).

Finally, in systems financed by fare revenue, the equilibrium fare is that which the fare revenue (paying users) is equal to the cost of the transport system in a given period. Thus, the higher the costs, the higher the value borne by the user directly. In a system of financing through public resources, the higher the costs, the greater the need for public resources (Carvalho & Pereira, 2011).

Thus, there is a conflict between fare and subsidy financing with regard to aspects such as social inclusion and urban development. Therefore, this study aimed to analyze, through cost functions, the economic viability of fare revenue of metro-rail transport companies, based on the application of the model with eight Brazilian metro-rail companies that represent 90% of passengers transported in the country's metro-rail system.

Thus, this research aims to analyze, through cost functions, the economic viability of fare revenue of metro-rail transport companies.

With this, it was possible to assess the values of fares, costs, and scale gains comparatively to establish the possible relationships for the viability of companies in terms of subsidy needs and fare values.

## 2 THEORETICAL FRAMEWORK

Cost concepts can be approached in different ways and by different theoretical conceptions. One of the distinctions usually made is the conception of costs in economics and accounting, referring almost always that accounting costs aim to record the facts, thus requiring a triggering event and a corresponding value, and economic costs are associated with opportunity costs. However, "on a theoretical level, given there is more than one school of thought in both Microeconomics and Management Accounting, the comparison between both would not be unique" (Panarella, 2010, p. 189).

Then, one can cover the cost analysis with different approaches, opting for the one that is more closely related to the research objective and, thus, allowing one to respond to the objective more robustly. Thus, as justified in the methodological procedures section, this article will be limited to explaining the one used for the analysis, restricted to the Microeconomic Theory of Cost Functions. Regarding the cost function, one should note that the economic cost criteria were used only to estimate the function since the basis for its application is accounting costs, and the cost function is based on the costs obtained through the financial statements. Thus, this section is subdivided into two subsections, one addressing cost function theory and the other that analyzes empirical studies that used this methodology to assess costs.

### 2.1 Cost functions

From an economic point of view, the "cost of an input is the remuneration that the remuneration the input would receive in its best alternative employment" (Nicholson & Snyder, 2018, p. 198), whose conception is called the opportunity cost of factors of production. Thus, to produce something, it is necessary to use the inputs necessary for this production and that, given the opportunity costs of the inputs, one will have the total costs of a certain level of product. The cost function is obtained by establishing a relationship between product levels and the minimum inputs for each of these levels (Eaton & Eaton, 1999).

Based on this description, then, the cost function is an implicit function of the production function, whose yields can be increasing, decreasing, or constant so that "there is a good relationship between the type of scale yield presented by the production function and the behavior of the cost function" (Varian, 2015, p. 392). Formally, then, the problem of cost minimization is now defined as (Mas-Colell et al., 1995, p. 139):

$$\begin{aligned} &\min_{z\geq 0} w * z \\ &Sj: \; f^*(z) \geq q \end{aligned} \tag{1}$$

Where $w$ is an input price vector, $z$ is a non-negative input vector, $f(z)$ is the production function, $q$ is the amount produced. The solution of the problem given in equation (1) gives the cost function defined as $c(w,q)$ and its conditional demand of the factor defined as $z(w,q)$. Considering only two inputs, the function $z(w,q)$ represents the cost line defined in the plane of the real numbers in $\Re^2$, which is the line that presents the same cost for all combinations of inputs, whose solution for "n" inputs is represented by the set $\{z \in \Re^L_+ : f(z) \geq q\}$, (where $L$ is the number of Lagrangians with $\ell$= 1,..., L-1) closed with respect to the origin. Being convex with respect to the origin, then, the first-order condition is necessary and sufficient so that:

$$w_l = \lambda \frac{\partial f(z^*)}{\partial z_l} \Rightarrow z_l^* > 0 \quad (2)$$

Where λ represents the marginal cost of production, that cost to be incurred when production varies by one unit. However, in the short term, some inputs will be fixed($z_f$), whose price vector is given by $w_f$ and other variables ($z_v$) with prices defined by $w_v$, so that the total cost (*TC*) of production will be given by a fixed portion (*FC*) and another variable (*VC*) with the volume of production, which is (Pindyck & Rubinfeld, 2014):

$$TC = FC + VC \quad (3)$$

When dividing the total costs by the quantity, the mean cost (*CMe*) is obtained. Then, when dividing both sides of equation (3), it can be verified that the *CMe* is equal to the sum of the mean fixed costs (*FCMe*) with the mean variable cost (*VCMe*):

$$TC/q = FC/q + VC/q \quad (4)$$

However, the demand for inputs ($z$) is dependent on the fixed structure, so the price vector can be established as $w = (w_f + w_v)$ and $z(w,q,z_f)$. Then, the cost curve geometry (*TC*) can be defined as (Varian, 1992):

$$c(w,q,z_f) = w_v z_v \left((w,q,z_f) + wfx_f\right) \quad (5)$$

And the mean cost, mean fixed cost, and mean variable cost as:

$$CMe = \frac{c(w,q,z_f)}{q} = \frac{w_f z_f}{q} + \frac{w_v z_v(w,q,z_f)}{q} \quad (6)$$

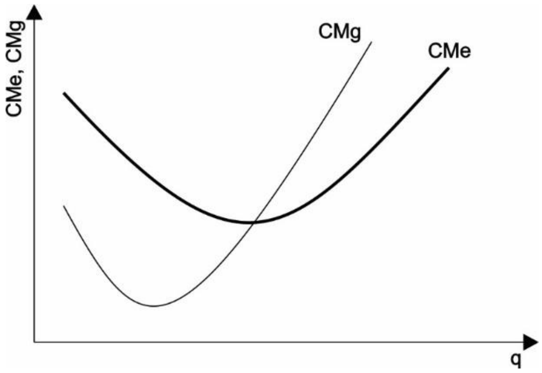
Usually, these functions derive from production functions with increasing yields for small production volumes or from using variable inputs in proportion to the fixed and decreasing ones from a given production volume. This results in short-term cost functions that a third-degree polynomial can represent:

$$Ct = \beta_0 + \beta_1 q + \beta_2 q^2 + \beta_3 q^3 \quad (7)$$

Where $\beta i$ represent the parameters of the function, having as a representation of fixed costs $\beta 0$ and, to represent a cost function adequately, must meet the following restrictions: $\beta_0 > 0, \beta_1 > 0, \beta_2 < 0\ e\ \beta_3 > 0$ (Besanko & Braeutigam, 2004).

Given that the mean cost is the division of the total cost by the amount produced, then this is usually a second-degree polynomial, representing a U-shaped mean cost curve with the marginal cost function (given in equation (2)) passing at its minimum point, as shown in Figure 1.

**Figure 1**
*Cost Curve Geometry*




Source: Frank (2013)

Notably, a large-scale firm may not obtain diminishing returns due to the magnitude of the market, which results in a mean cost function in an "L" format or presenting only the decreasing segment of the function drawn in Figure 1. Finally, a firm would only make a profit if its total unit revenue was higher than the mean costs of the cost function. Considering the sales prices (P) constant for any production level (q), the profit would be maximized when the price was equal to the Marginal Cost, that is, $P = MCg$ *and* $P > CMe$.

## 2.2 Empirical studies

The empirical studies analyzed in this article were selected from a search on the Capes/MEC Journal Portal website with the keywords "economies of scale", "cost function", and "transport". These studies reveal different applications and approaches of the cost function and analysis of economies of scale. As Hörcher and Tirachini (2021) refer, the estimation of cost functions has a long history in the transport economy since cost estimation is a relevant element for assessing policies. Also, the authors point out that defining the cost function and the appropriate measures of economies of scale is not a simple task for a transport operator since selecting a production measure is a key point when estimating a cost function.

Concerning economies of scale, there are relevant implications for the economic assessment of transport systems from scale gains, highlighting the study by Anupriya et al. (2020), having as methodology the short-term cost functions for metropolitan metro-rail systems from different locations in the world, concluding that systems with high usage density are the most economical and that operating costs decrease as the size of the subway increases. Furthermore, the authors found a high disparity in the operational costs of metro-rail systems. In the research by Anupriya et al. (2020), the variables used in the regression model were total operating costs, whose main components were service costs, administration costs, and maintenance costs.

Analyzing the economies of scale in air transport, based on data from the largest American airlines, the results show that, in this segment, the gains in scale are modest and, based on this result, the scale does not impact the results arising from the fares (Johnston & Ozment, 2013). And, in transporting passengers by bus, to estimate the cost function, Singh (2014) assumes that companies minimize the cost subject to a production function, identifying that technological progress is neutral and there are significant gains in scale.

However, Matas and Raymond (1998), in a study conducted on public transport in Spain, an oligopolized market, inferred that returns to scale are constant, with diseconomies for large companies. In this context, the authors state that introducing competition in the market could considerably reduce operating costs.

Batarce and Galilea (2018) add in the analysis that the fare price differs from the studies mentioned above. Their findings point to economies of scale and identify that the subsidy for bus transportation in Santiago/Chile is justified, given that the actual fare is lower than the equilibrium fare.

Besides the empirical studies that aim to identify economies of scale in the general context of the transport system, the use of the cost function is identified to analyze economies of scale related to agglomeration cost in vehicles for different modes of transport (Qin, 2014), impact of congestion (Ying & Yang, 2005; Fernandez et al., 2005), road concessions (Vergara-Novoaet al., 2020), and cost of road renewal (Link, 2006). The studies by Basso and Jara-Diaz (2006), Cambini et al. (2007), and Ayadi and Hammani (2015) assess economies of scale related to the expansion of transport networks.

From the perspective of efficiency, studies such as Martini et al. (2020) analyzed the Italian airport system and, among the findings, identified that low-cost airlines reduce short- and long-term inefficiencies of the system, in addition to evidence of economies of scale. Botasso et al. (2019), with reference to the UK airport industry, state that economies of scale are important only for airports of up to about 5 million passengers since larger airports tend to operate with approximately constant returns to scale.

Still on efficiency, Obeng and Sakano (2020) studied the effects of regulations, input subsidies, their interactions, and cost efficiency in transportation in the United States. They show how a firm's cost efficiency relates to society's cost efficiency. Finally, Liu (2021), using the duality between production and cost functions, proposed a new approach to estimate cost inefficiency applicable to general cases. In conclusion, the author reports that the results of the proposed solution can identify the inefficiency of each input and derive robust estimates of economies of scale, which are important for policy implications and management decisions.

Expanding the research conducted, it is identified there are gains in scale in sectors such as Education, Insurance, Telecommunication, and Health. The estimation of these gains is made through cost functions, as in the studies by Agasisti and Jhones (2016), Klotzki et al. (2018), Vendrúsculo and Alves (2009), Gomez et al. (2020) and Chattopadhyay (2021). Although these segments differ from what is addressed in this study, the procedure for detecting scale occupancy gains was the same through cost functions. According to Vendrúsculo and Alves (2009), the cost curves show the variation of the total cost to the production level. Through the short-term total cost curve, it is possible to identify the optimal combination of production that minimizes costs. Furthermore, the authors infer that through the economy of scale, it is possible to maximize companies' profit as the production level increases.

## 3 METHODOLOGICAL PROCEDURES

This section describes the companies, followed by the model, ending with data source and processing.

### 3.1 Brazilian metro-rail companies

This study included eight medium and large metro-rail companies, identified in Table 1, representing 90% of passengers transported in the Brazilian system, five public and three private. Data collection took place from 2014 to 2019, in order to exclude possible distortions caused by the effects of the pandemic.

**Table 1**
*Brazilian metro-rail companies*

| Acronym | Firm | State | Type |
|---|---|---|---|
| Metrô SP | Companhia do Metropolitano de São Paulo. | São Paulo | Public |
| CPTM | Companhia Paulista de Trens Metropolitanos. | São Paulo | Public |
| VIAQUATRO | Concessionária da Linha 4 do Metrô de São Paulo S.A. | São Paulo | Private |
| Metrô RIO | Concessão Metroviária do Rio de Janeiro S.A. | Rio de Janeiro | Private |
| SUPERVIA | Supervia Concessionária de Transporte Ferroviário S.A. | Rio de Janeiro | Private |
| CBTU BH | Companhia Brasileira de Trens Urbanos. | Minas Gerais | Public |
| TRENSURB | Empresa de Trens Urbanos de Porto Alegre S.A. | Rio Grande do Sul | Public |
| Metrô DF | Companhia do Metropolitano do Distrito Federal. | Brasília | Public |

Source: Prepared by the authors.

### 3.2 Estimation Model

The estimated model was based on that presented in equation (7), but estimated for mean costs, following the models already established in the studies by Wilson (1981) and Koshal and Koshal (1995). The mean cost estimation adds two contributions to the total costs of companies. The first refers to the constant. Since the data have values very far from the origin, there would be a very wide distortion to consider it represented the fixed costs. The second, when considering the mean costs, there is a reduction in the discrepancy of values. Thus, the estimated model was:

$$CMe_t = \beta_0 + \beta_1 q_t + \beta_2 q^2_t + \varepsilon_t \tag{8}$$

Where $\beta_i$ are the parameters to be estimated, $q_t$ is the number of paying passengers transported, in millions, in month "$t$" and $CMe_t$ is the mean monthly operating cost in reais, $\varepsilon_t$ is the error that represents the mean effect, for each level of passengers transported, of all variables that impact the cost and that are not in the model.

The mean cost was calculated based on the division of the total cost by the number of passengers transported, where the total cost is the sum of the costs of the services provided and general and administrative expenses.

As for the expected signals of the estimated betas, with regard to the theoretical consistency of the results, it is that $\beta_0 > 0,\ \beta_1 < 0,\ \beta_2 > 0$, and the robustness of the model is associated only with the level of significance of the betas, the coefficient of determination, the betas signals, and the stability test specified according to Maddala (2003) as:

$$F = \frac{SQRR - SQRI/(k+1)}{SQRI(n1 + n2 - 2k - 2)} \tag{9}$$

Where SQRR is the sum of regression squares with all sample data, SQRI is the sum of unrestricted regression squares (composed of the sum of squares of the various sample clusters), k is the number of regressors, and $n_i$ is the sample set size (in the case of equation (9) with the constitution of two subsets from the total sample).

The model given in equation (8) was estimated by Ordinary Least Squares, considering this is the expected theoretical model. There is no collinearity, given that this is a phenomenon of linear relationships, and the variable enters at the level and frame (Gujarati, 2006). Autocorrelation is a natural and expected element for this type of relationship. With autocorrelation, the estimators by OLS are still linear and unbiased, but are not more efficient, as they are not the ones with minimum variance (Pindyck & Rubinfeld, 2004). However, this property is not required for the analysis because there is no hypothesis testing or value prediction.

The Efficiency Frontier of cost minimization points was estimated by Ordinary Least Squares from the frontier formed by the minimum value errors of the equation estimation (8).

### 3.3 Period, source, and processing of data

The data refer to the number of passengers carried by the sample companies, defined as *q* in the model (8), and their operating costs from 2014 to 2019. The operating costs were calculated by adding the costs of the services provided and the general and administrative expenses in the financial statements published on the websites of the companies in the sample. For years when information was unavailable, it was obtained from companies through external service channels.

With regard to costs, current monetary values were inflated by the General Price Index - internal availability (IGP-DI) of the Getúlio Vargas Foundation, available at www.portalbrasil.net., thus composing the estimate in constant values.

## 4 RESULTS

This section is subdivided into two subsections. The first presents briefly data on the costs of the Brazilian metro-rail companies in the sample, and the second contains the regression results.

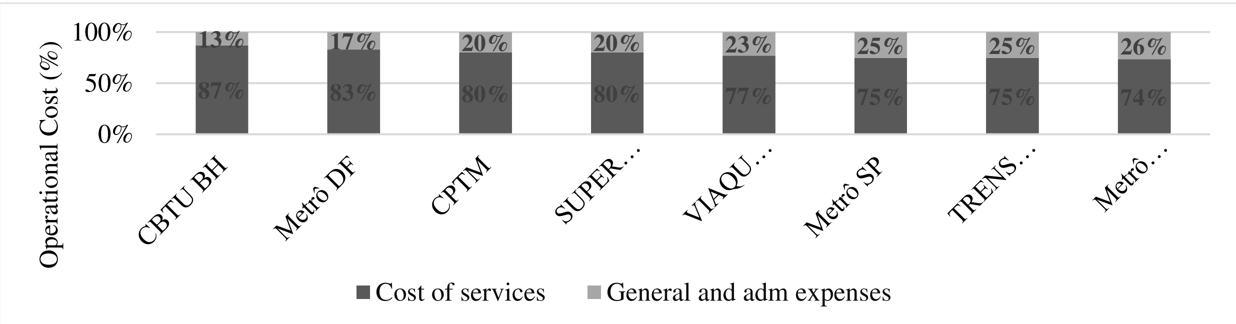
### 4.1 Costs of the metro-rail sector

The processes can be classified into transport and support services in the metro-rail system. Transportation services are divided into primary and secondary. The primaries constitute the operating system; they are directly related to the transport user and are composed of operations in the box office, operational safety, train operation, and traffic control. Secondary transportation services involve human resources, equipment, and facilities for operating system maintenance. Support services represent expenses with general administration and involve planning, financial administration, supplies, human resources, information technology, among others (Pezerico, 2002). Primary and secondary transport services constitute the cost of the services provided. In contrast, support services are represented by general and administrative expenses, which constitute the firm's operating cost.

Figure 2 shows the composition of the costs of the services provided and the general and administrative expenses in forming the total operating cost of the metro-rail companies in the sample.

**Figure 2**

*Mean representativeness of the costs of services provided and general and administrative expenses incurred from 2014-2019*




Source: Prepared by the authors.

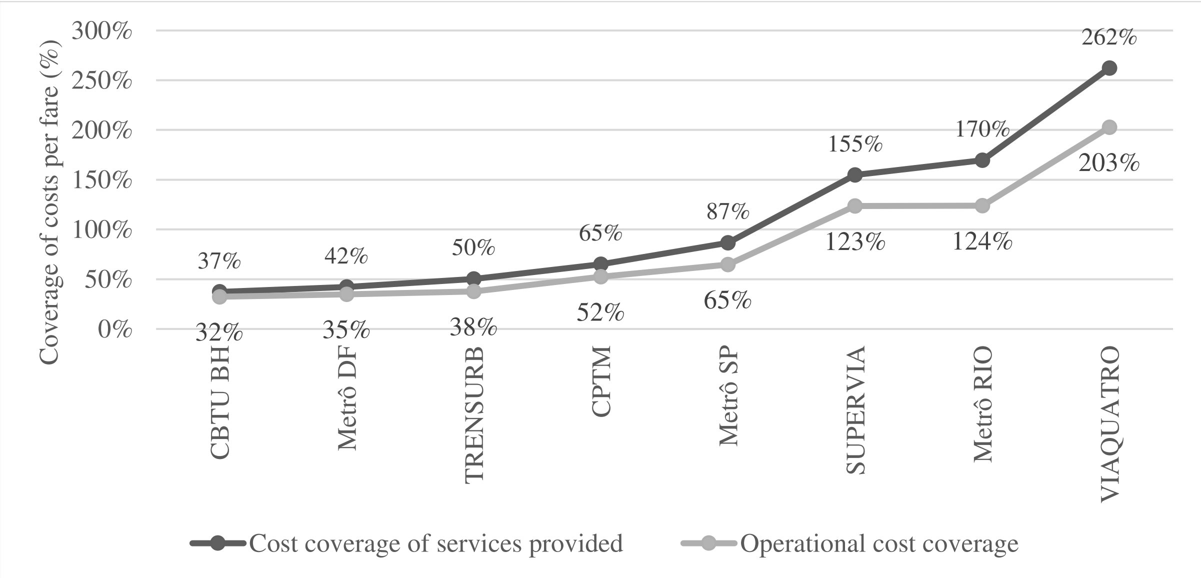
According to Figure 2, the cost levels of the services provided relative to the operating cost vary from 74% (Metrô Rio) to 87% (CBTU BH), and the costs of personnel, third-party services, and electricity, on average, represent 76% of the cost of the services provided. Personnel costs are the most representative in systems, followed by third-party services costs. These two are related insofar as the level of outsourcing of services replaces the use of own labor. In this context, Metrô SP stands out, in which the predominance of using own labor in maintenance services reflects the lower cost of third-party services. Electricity, the main input for the operation of the metro-rail transport system, varies from 6% to 18% and is directly impacted by how metro-rail transport companies acquire this energy. In addition to the form of acquisition, other factors, such as the different electricity fares by region and the technology of the trains in operation, affect the cost of transport operators.

As for the cost of services provided, it is important to highlight the particular situation of public companies dependent on government resources, in which the need for resources can affect, for example, the level of contracting services, with a lower cost given the unavailability of budget for contracting. Therefore, the lower cost is not a reflection of efficiency.

Figure 3 shows the level of coverage of the cost of services and the operating cost by fare revenue, ranging from 32% to 203% for the operating cost and from 37% to 262% for the cost of services provided. The highest levels of coverage of costs by fare revenue are obtained in companies operating through private concession, followed by the two public companies that concentrate the highest level of passengers transported, CPTM and Metrô SP. As for the fare, it should be noted that its determination in the transport system reflects the public policy adopted. This policy defines the level of financing of costs by the transport user and the level of financing by government resources. In other words, it determines the financing model for transport costing.

**Figure 3**

*Level of coverage of costs by fare revenue*

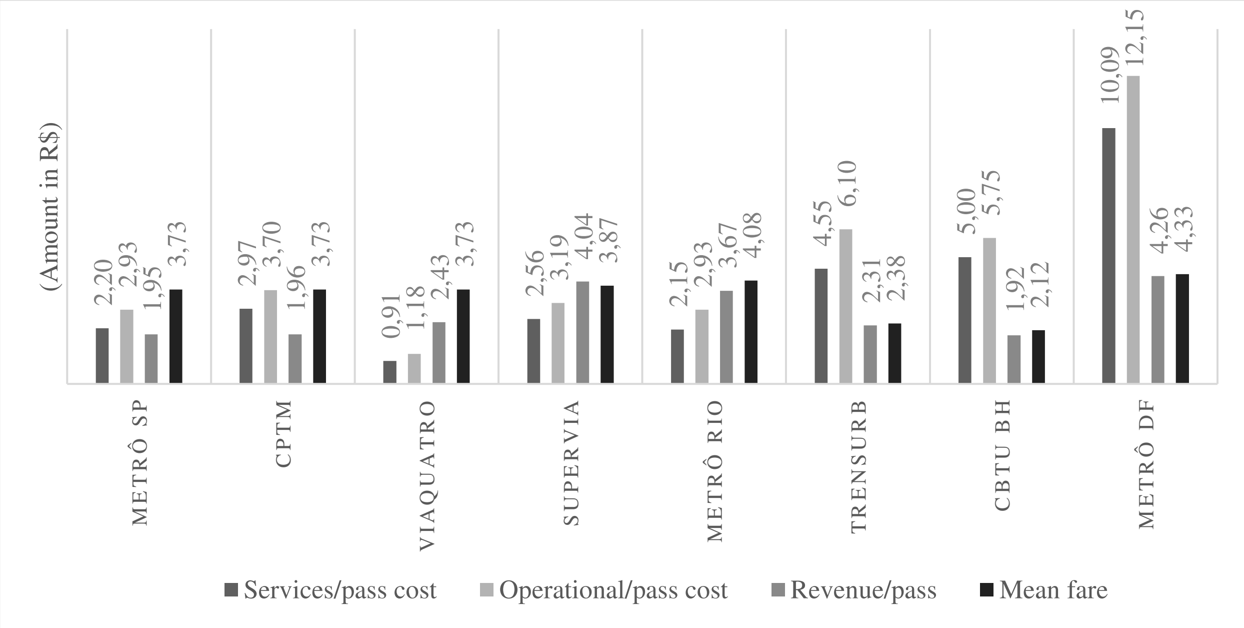



Source: Prepared by the authors.

Figure 4 shows the mean fares charged by metro-rail companies compared to the mean costs and revenues per passenger from 2014 to 2019.

**Figure 4**

*Costs, revenues, and mean rates 2014 – 2019*




Source: Prepared by the authors.

There are relevant differences between revenue per passenger and the unit fare charged. One factor contributing to these differences is the integration policies between the modes of transport, which generate a reduction in the full unit fare, given the benefit of integration to the paying user, in addition to passengers benefiting from public policies of gratuities and subsidies.

## 4.2 Analysis of the cost functions and fare viability of Brazilian metro-rail transport

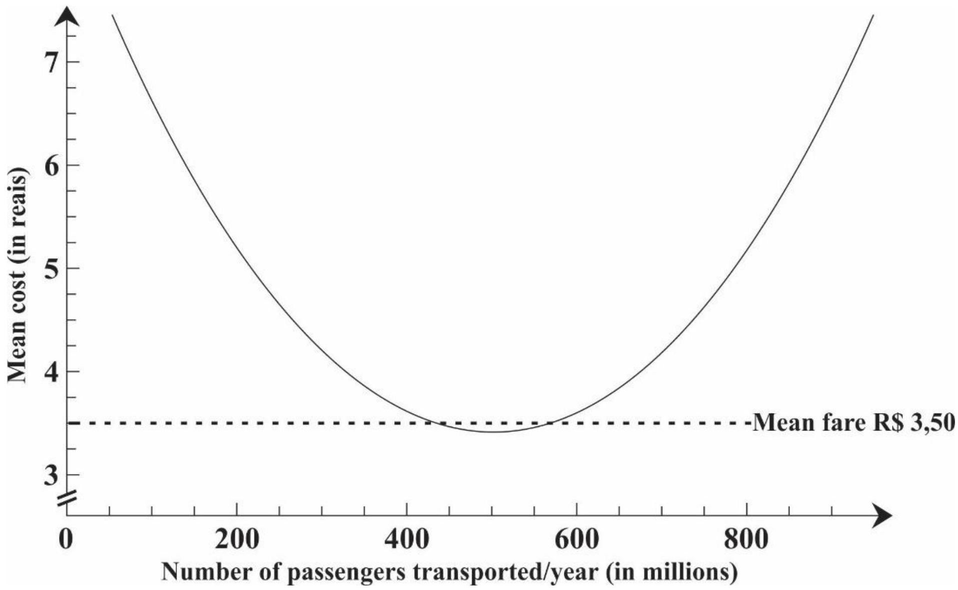
The results of the estimation of the model proposed in equation (8) generated the following results (in parentheses, below the estimator, is its respective standard deviation):

$$\begin{array}{c} CMe_t = 2E\text{-}0.5x^2 \quad - 0.02x \quad + 8.5 \\ (4.57\ E\text{-}05)\ (0.005150)\ (0.734762) \end{array} \tag{10}$$

The regression presented the adjusted coefficient of determination of 0.432, a value of low power of explanation for the model, influenced to a large extent by the high dispersion of mean costs. The dispersion of the structures can explain this fact in terms of the metro-rail network operated by the companies, which generates significantly different values in the number of passengers transported. Nevertheless, the signals of the estimated parameters meet what is theoretically expected and are significant at 1%. The Durbin-Watson test indicated positive autocorrelation (DW= 0.038), indicating the existence of high costs in a given period will result in high costs in the following period.

Given the significance of the results, the values obtained are interpreted. To synthesize and better visualize the results, Figure 5 was built. It allows visualizing the format of the mean cost curve and the use of installed capacity in terms of costs.

**Figure 5**
*Mean cost function*




Source: Prepared by the authors.

In a general analysis of Figure 5, it is verified that initially, there is an economy of scale. That means that, as the number of passengers increases, there is a reduction in the mean cost. Therefore, there is a gain of scale, with companies below the optimal scale. This situation is verified by up to 500 million passengers; from then on, the increase in the number of passengers reflects an increase in the mean cost.

However, these results should be carefully analyzed, as the data has a dichotomy. There is a set of companies with a small number of passengers transported and another with a high number of passengers. This element detects a weakness in establishing the regression function for intermediate values so that the central values should not be interpreted as being reliable, but rather understand there is a strong gain in economies of scale to increase the number of passengers transported initially and, after a certain volume, there is an increase without a defined format. Thus, it is impossible to establish the minimum point's value and in what magnitude of passengers this would occur.

The companies located to the right in Figure 5, those with more than 500 million passengers transported per year, are the companies CPTM and Metrô SP, both public companies, which concentrate the largest volume of passengers transported in the Brazilian metro-rail system. These companies operate above their optimum point, and a reduction in scale would reduce their mean costs. The other metro-rail companies are to the left of the graph, in the range of fewer than 500 million passengers transported/year, and operate below their production capacity.

From the above, it is possible to state that Brazilian metro-rail companies do not benefit from gains in scale, and it can also be inferred that this reflects directly on the cost of transport services. Shedding light on the fare issue, it should be noted that the mean unit fare from 2014 to 2019 was R$3.50, and the mean cost per passenger was R$3.38. However, considering the effect of public subsidy policies that exempt or reduce transport fares, as well as integration policies with other modes of transport that reduce the value of the unit fare, the mean fare revenue per passenger was R$2.31, reflecting, in these data, the dependence on government resources to subsidize the cost of the operations of this mode of transport. Thus, it can be said that even optimizing costs, that is, operating at its optimum point, there is a deficit in fare revenue in the Brazilian metro-rail system. On average, there is a loss per passenger due to not entirely using the installed capacity.

## 5 CONCLUSIONS

Applying the model allowed us to verify that the mean fares practiced by public companies cannot generate fare sustainability, presenting a high deficit from the point of view of fare revenue, which generates the need for subsidies to maintain their operations and realize investments.

However, verifying significant gains in occupation or using the current scale was also possible, reducing the dependence on subsidies. These could be even smaller if companies could operate permanently in their efficiency frontiers. In this case, operating at their optimum cost point, companies would be marginally in equilibrium, and a change in fares could make them surplus in terms of fares and service costs. However, this possibility encounters two difficulties: the first refers to demand - there should be a recomposition of household income to compensate for the increase in the fare and increase the demand for train travel in the regions where they operate. The second is that, given the diversion, it would be difficult to consider that the firm could permanently operate at its maximum efficiency point. Nonetheless, this result indicates possible movements for companies that would significantly improve their dependence on subsidies.

As future studies, it is suggested to expand this research through qualitative and case studies, given the relevance and impact of the relationship between operating cost and fare revenue for metro-rail companies.

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